# Magnetic Pair Spectrometer Studies of Electromagnetic Transitions in $Be^{10}$ and $B^{10}$ <sup>†</sup>

E. K. WARBURTON, D. E. ALBURGER, AND D. H. WILKINSON\* Brookhaven National Laboratory, Upton, New York (Received 3 June 1963)

An intermediate-image pair spectrometer was used to study electromagnetic transitions in Be<sup>10</sup> and B<sup>10</sup>. Energy levels in these nuclei were populated by means of the  $Be^{9}(d,p)Be^{10}$  and  $Be^{9}(d,n)B^{10}$  reactions with deuteron energies between 2.0 and 3.2 MeV. In Be<sup>10</sup> the branching ratios of the  $5.96 \rightarrow 0$  and  $5.96 \rightarrow 3.37$ transitions were determined to be  $48\pm2\%$  and  $52\pm2\%$ , respectively. A pair line corresponding to the  $6.26 \rightarrow 3.37$  transition was observed and an upper limit of 0.4% was placed on the relative intensity of the  $6.26 \rightarrow 0$  transition which supports an assignment of 2<sup>-</sup> for the Be<sup>10</sup> 6.26-MeV level. In addition, a  $6.18 \rightarrow 0$ transition was observed but not a  $6.18 \rightarrow 3.37$  transition. The energy difference between the  $6.18 \rightarrow 0$  pair line and the  $5.96 \rightarrow 0$  pair line was measured with sufficient accuracy to show that the Be<sup>10</sup> 6.18-MeV level has a mean lifetime greater than  $5 \times 10^{-13}$  sec, and consequently has a most probable spin-parity assignment of 0<sup>+</sup>. In B<sup>10</sup> the relative intensities of the  $3.58 \rightarrow 0.72$  and  $3.58 \rightarrow 0$  transitions were found to be  $(4.2\pm0.3)$ :1 if both transitions are M1 while the branching ratios of the  $5.16 \rightarrow 2.15$ ,  $5.16 \rightarrow 0.72$ , and  $5.16 \rightarrow 0$  transitions were found to be  $(65\pm2)\%$ ,  $(29.5\pm2)\%$ , and  $(5.5\pm0.7)\%$ , respectively. Transitions from the B<sup>10</sup> 4.77-MeV level were not observed and a limit of  $\Gamma_{\gamma}/\Gamma < 0.05$  was set for this  $\alpha$ -particle unbound level. Data are presented to enable  $\Gamma_{\gamma}/\Gamma$  for the 5.16-MeV level of B<sup>10</sup> to be computed when the relative production cross sections of that state and the 3.37-MeV state of Be<sup>10</sup> in deuteron bombardments become known. In order to obtain these results, calculations of the spectrometer pair-line efficiency were extended to include the emission of internal pairs from aligned nuclei. The experimental results are compared to the predictions of the independent-particle model.

#### INTRODUCTION

HE magnetic-lens intermediate-image pair spectrometer<sup>1-3</sup> designed and built at this laboratory has properties which make it a valuable tool for the investigation of electromagnetic transitions with energies above about 2 MeV. Among the more important of these properties are: (1) High luminosity (defined as the product of the transmission and the source area); (2) a resolution (full width at half-maximum) continuously variable from 0.5 to 3%, and (3) the fact that with this instrument nuclear or internal pairs produced by accelerator bombardment of a target can be observed just about as easily as radiations from a radioactive source.

Several investigations<sup>1-5</sup> have been made at this laboratory of electromagnetic transitions initiated by a Van de Graaff accelerator. Somewhat more extensive studies<sup>6-10</sup> of accelerator-induced transitions have been

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carried out at Rice University with an instrument of the same general type although the Rice spectrometer has a considerably lower luminosity than the Brookhaven spectrometer. Recently, the usefulness of the Brookhaven spectrometer has been enhanced by a detailed calculation of the efficiency<sup>5</sup> of this instrument as a function of transition energy for E0 and the first four orders (i.e., l=1 to 4) of El and Ml radiation. These calculations were made for the case in which there is no preferred direction in space provided by the nuclear reaction initiating the pair emission, i.e., the accompanying gamma rays are emitted isotropically. In the general case, however, the reaction which forms the nucleus in an excited state will cause an alignment so that an axis of rotational symmetry is established. i.e., the accompanying gamma rays will not, in general, be emitted isotropically. In the next section the efficiency calculations of Wilkinson et al.5 are extended to cover the case of emission of internal pairs from aligned nuclei. These calculations make it possible to obtain transition branching ratios with an accuracy which, in favorable cases, is considerably better than can be obtained by other means.

The present work is an investigation of the transitions following the bombardment of Be<sup>9</sup> by deuterons with energies between 2.0 and 3.2 MeV. For these bombarding energies the only transitions expected with energies greater than 2 MeV, aside from very weak  $Be^{9}(d,\gamma)B^{11}$  transitions, are those from the  $Be^{9}(d,p)Be^{10}$ (Q=4.59 MeV) and  $Be^{9}(d,n)B^{10}$  (Q=4.36 MeV) reactions. Previous studies<sup>11</sup> of transitions following  $Be^9+d$ have included investigations<sup>8,10</sup> using the Rice pair

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<sup>\*</sup> Permanent address: Nuclear Physics Laboratory, Oxford, England.

<sup>&</sup>lt;sup>11</sup> F. Ajzenberg-Selove and T. Lauritsen, Nucl. Phys. 11, 1 (1959).

spectrometer. However, because of the higher luminosity and inherently better resolution of the Brookhaven spectrometer we felt that another investigation of the transitions from  $Be^9+d$  would be worthwhile. Our major reasons for initiating this work were to gain further information on the B10 5.16-MeV level and to search for a ground-state transition from the Be<sup>10</sup> 6.18-MeV level.

Just before this work was begun the mystery surrounding the properties of the B<sup>10</sup> 5.16-MeV level (see, e.g., Warburton and Chase<sup>12</sup>) was cleared up by the discovery of the broad B<sup>10</sup> 5.18-MeV level<sup>13</sup> with  $J^{\pi}$ ,  $T=1^+, 0.1^4$  Since that time the T=1 assignment to the 5.16-MeV level has been verified.<sup>15</sup> Thus, there is no longer any reason to doubt the original identification<sup>16</sup> of the 5.16-MeV level as the  $J^{\pi}$ ,  $T=2^+$ , 1 analog of the Be<sup>10</sup> first excited state. In fact, the recent Be<sup>9</sup> $(d,n\gamma)$ B<sup>10</sup> work of Garg, Gale, and Calvert<sup>17</sup> may be combined with earlier work, to make it very probable that the 5.16-MeV level is indeed  $J^{\pi}$ ,  $T=2^+$ , 1. The arguments leading to this conclusion are given below.

The angular distributions<sup>18</sup> of gamma rays in the reaction  $Li^{6}(\alpha, \gamma)B^{10}$  are consistent with assignments of  $J^{\pi} = 1^{-}, 1^{+}, \text{ or } 2^{+} \text{ to the } B^{10} 5.16$ -MeV level. Odd parity is excluded by the l=1 stripping pattern found<sup>17,19</sup> in  $Be^{9}(d,n)B^{10}$  and recent studies<sup>20</sup> of the internal pairs associated with the B<sup>10</sup> 5.16  $\rightarrow$  0.72 transition confirm the even parity assignment. If the 5.16-MeV level were 1<sup>+</sup>, then the Li<sup>6</sup>( $\alpha,\gamma$ )B<sup>10</sup> reaction cross section<sup>18</sup> leads<sup>12,21</sup> to lower limits on  $\Gamma_{\gamma}$  of 0.35 and 0.16 eV for the  $5.16 \rightarrow 2.15$  and  $5.16 \rightarrow 0.72$  transitions, respectively. The former is large enough to rule strongly in favor of  $T = 1.^{21}$  We can use these lower limits to the  $\Gamma_{\gamma}$  to obtain upper limits for  $x^2$ , the intensity ratio of E2 to M1 radiation. We take  $Z^2$  times the Weisskopf estimate<sup>22</sup> (using a radius constant 1.2 F) as a reasonable upper limit to the E2 radiative width. We remark that this is quite conservative since E2 transitions in self-conjugate nuclei which change the isotopic-spin are not expected to have collective enhancement<sup>23</sup> of the kind normally found between low-lying states of light nuclei. Combining these upper limits on  $\Gamma_{E2}$  with the lower limits on  $\Gamma_{\gamma}$ 

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gives upper limits on  $x^2$  of 0.02 and 0.28 for the B<sup>10</sup>  $5.16 \rightarrow 2.15$  and  $5.16 \rightarrow 0.72$  transitions, respectively. Without going into details, the Be<sup>9</sup> $(d,n\gamma)$ B<sup>10</sup> correlation experiments of Garg et al.<sup>17</sup> essentially demand that for a 1<sup>+</sup> assignment to the B<sup>10</sup> 5.16-MeV level  $x^2$  lies approximately in the range  $0.36 \le x^2 \le 3.0$  for the  $5.16 \rightarrow 2.15$ transition. Otherwise, the anisotropy calculated from plane-wave stripping theory is not as large as that observed experimentally. As remarked by these authors, the anisotropy calculated using plane-wave theory is expected to be an upper limit. Thus, the inconsistency between the upper limit of 0.02 for  $x^2$  demanded by the radiative width and the requirement  $0.36 \le x^2 \le 3.0$ from the correlation experiment clearly rules out  $J^{\pi} = 1^+$  with the result that  $J^{\pi}$ ,  $T = 2^+$ , 1 is left as the only possible assignment for the B<sup>10</sup> 5.16-MeV level.

The radiative widths of the gamma-ray transitions from the lowest  $J^{\pi}$ ,  $T=2^+$ , 1 state in  $B^{10}$  have been predicted<sup>24</sup> on the independent-particle model (IPM) and have been found to be in poor agreement with those of the B<sup>10</sup> 5.16-MeV level.<sup>12</sup> In view of the virtual certainty of the  $2^+$  T=1 assignment it is of interest to check the experimental values for the radiative widths of this level. The isotopic spin-mixing situation would be illuminated by a measurement of the alpha-particle width of this level. The reaction  $Li^{6}(\alpha, \gamma)B^{10}$  gives the usual information about the combined alpha-particle and gamma-ray widths. The two may be disentangled by a measurement of the alpha-gamma branching ratio. The object of the present work was then to gain an accurate measurement of the gamma-ray branching ratios and to obtain information bearing on the alphagamma branching ratio in order to provide material for the disentangling of  $\Gamma_{\alpha}$  and  $\Gamma_{\gamma}$ .

We may note at this point however, that the disagreement between the experimental branching ratios from this level as reported in the literature and those predicted by the IPM is, of course, unaffected by the determination of the absolute widths.

In view of the poor performance of the IPM on the gamma widths of the 5.16-MeV level its success (or otherwise) for other states of A = 10 nuclei is especially interesting. A more accurate determination of the branching ratios of the  $2^+$  T=0 level at 3.58 MeV in  $B^{10}$  is welcome since the current figures in the literature indicate disagreement with the IPM here also.

The Be<sup>10</sup> 6.18-MeV level is excited quite weakly by the  $Be^{9}(d,p)Be^{10}$  reaction<sup>11</sup> and radiative transitions from this level have not been observed. This level has been predicted to be<sup>25</sup>  $J^{\pi} = 0^+$  so that one aim of this investigation was to search for a  $6.18 \rightarrow 0$  transition and to study its properties. A related interest in finding the assignment for this level concerns the effective values for the parameters of the IPM in this part of

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<sup>&</sup>lt;sup>22</sup> D. H. Wilkinson, in *Nuclear Spectroscopy*, edited by F. Ajzenberg-Selove (Academic Press Inc., New York, 1960), Part B, pp. 852–889. <sup>29</sup> E. K. Warburton, Phys. Rev. Letters 1, 68 (1958).

<sup>24</sup> D. Kurath, Phys. Rev. 106, 975 (1957).

<sup>&</sup>lt;sup>25</sup> W. W. True and E. K. Warburton, Nucl. Phys. 22, 426 (1961).

the 1*p*-shell. Some evidence<sup>26</sup> from A = 13 suggests that a/K may be considerably lower ( $\approx 3$  or less) than previously supposed ( $\approx 5$ ). A recent investigation<sup>27</sup> of E2 lifetimes in Be<sup>10</sup> and B<sup>10</sup> makes  $a/K \approx 3$  for A = 10rather unlikely. One aspect of this situation is that at  $a/K \approx 3$  the second 2<sup>+</sup> state of Be<sup>10</sup> has come down to approximately the same excitation as the first 2<sup>+</sup> state.<sup>28</sup> Apart from the states at 5.96 and 6.26 MeV, which are of odd parity, the only state below 7.37 MeV (itself of J=3) is that at 6.18 MeV. If it could be shown that this is not  $2^+$  the first candidate for the second  $2^+$  state of the IPM would become that at 7.54 MeV (which indeed has J=2). This would argue strongly against a low value of a/K for the A = 10 system unless this state is peculiarly sensitive to other parameters of the model. (In the version presented by Kurath<sup>28</sup> the demonstration that the 6.18-MeV state is not 2<sup>+</sup> implies  $a/K \ge 5.8$ -the equality corresponding to the identification of the IPM state with that at 7.54 MeV.)

## II. THE PAIR SPECTROMETER EFFICIENCY

Rose<sup>29</sup> calculated the probability of internal pair formation relative to gamma emission and the electronpositron angular correlation for the case of nonaligned nuclei (i.e., equal populations of the  $m_i$  substates of the emitting level). Rose's results were used in the previous calculation of the spectrometer efficiency.<sup>5</sup>

Since most of the transitions initiated by nuclear reactions are from nuclei with some degree of alignment, it is desirable to know the effects of alignment on the spectrometer efficiency. Recently, the results of Rose<sup>29</sup> and Goldring<sup>30</sup> have been extended to cover the general correlation function describing the emission of internal pairs from aligned nuclei.<sup>31</sup> We shall use these results to calculate the efficiency of the spectrometer for the case that the  $m_i$  substates of the emitting level are not equal and for interference between Ml and El+1radiation.

The emission of internal pairs is described<sup>30</sup> by the angular correlation function,  $F(\theta, \delta, \theta_q, \phi_q)$ , where  $\theta$  is the angle between the positron and electron directions,  $\theta_q$  and  $\phi_q$  are the polar and azimuthal angles of  $\mathbf{q} = \mathbf{p}_+$  $+p_{-}$ , and  $\delta$  is the dihedral angle formed by the planes  $(\mathbf{z},\mathbf{q})$  and  $(\mathbf{p}_+,\mathbf{q})$  where  $\mathbf{z}$  is a vector in the direction of the axis of quantization (beam axis). Then  $F(\theta, \delta, \theta_q, \phi_q) d\Omega_+ d\Omega_- dW_+$  is the ratio of the number of pairs emitted per second into solid angles  $d\Omega_+ d\Omega_-$  and with the positron energy between  $W_+$  and  $W_++dW_+$ to the number of gamma rays emitted per second.

For our geometry  $\delta = \frac{1}{2}\pi$  and the positron and electron

energies and momenta are equal, i.e.,  $W_+ = W_- = \frac{1}{2}k$ ,  $p_+ = p_- = (\frac{1}{4}k^2 - 1)^{1/2}$ , where k is the transition energy in units of the electron rest mass. Also, for nonpolarized beams and targets, the beam axis is an axis of rotational symmetry so that  $F(\theta, \delta, \theta_q, \phi_q)$  is independent of  $\phi_q$ . For these conditions the correlation function for pure *Ml* or *El* radiations becomes

$$8\pi^{2}F_{l}(\theta, \frac{1}{2}\pi, \theta_{q}) = \sum_{\nu} A_{\nu}(ll) [P_{\nu}(\cos\theta_{q})\gamma_{l}(\theta)$$

$$-(-)^{\sigma}\kappa_{\nu}(ll)P_{\nu}^{(2)}(\cos\theta_{q})L_{l}(\theta)]$$

$$-\sigma(2l/l+1)(q/k)^{2l-4}[(q/k)^{2}\gamma_{E1}(\theta)-\gamma_{M1}(\theta)]$$

$$\times \sum_{\nu} A_{\nu}(ll) \{1+[C(ll\nu;00)/C(ll\nu;1,-1)]\}$$

$$\times P_{\nu}(\cos\theta_{q}), \quad (1)$$

where  $\sigma = 0$  for *Ml* transitions and  $\sigma = 1$  for *El* transitions,  $P_{\nu}^{(2)}(\cos\theta_q)$  is an unnormalized associated Legendre polynomial, and  $\kappa_{\nu}(ll)$  is given by

$$\kappa_{\nu}(ll') = -\left[\frac{(\nu-2)!}{(\nu+2)!}\right] \frac{C(ll'\nu;11)}{C(ll'\nu;1,-1)},$$
(2)

with l'=l. The sum is for  $\nu$  even from  $\nu=0$  to  $\nu=2l$ . The  $C(ll'\nu; 00)$ , etc., are the familiar vector-addition coefficients. The  $\kappa_{\nu}(ll')$  are tabulated by Fagg and Hanna<sup>32</sup> and  $\gamma_l(\theta)$ , which is the correlation function integrated over all angles except  $\theta$ , is given by Rose,<sup>29</sup>  $\gamma_{M1}(\theta)$  and  $\gamma_{E1}(\theta)$  are the  $\gamma_{l}(\theta)$  for pure magnetic dipole and pure electric dipole transitions, respectively. The quantity  $L_l(\theta)$  is given by

$$L_{l}(\theta) = (2/137\pi k^{3})(\frac{1}{4}k^{2}-1)^{3} \\ \times [\sin^{2}\theta/(k^{2}-q^{2})^{2}](q/k)^{2l-2\sigma-2}.$$
(3)

For a mixed Ml, El+1 transition the correlation function has the form<sup>31</sup>

$$F_{M,E} = \frac{F_l + 2xF_{l,l+1} + x^2F_{l+1}}{1 + x^2}, \qquad (4)$$

where  $x^2$  is the ratio of the intensities of El+1 to Mlradiation for the associated gamma rays. For our geometry, the interference term,  $F_{l,l+1}$ , is given by

$$8\pi^{2}F_{l,l+1}(\theta, \frac{1}{2}\pi, \theta_{q}) = \sum_{\nu} A_{\nu}(\mathcal{U}') [P_{\nu}(\cos\theta_{q})\gamma_{Ml}(\theta) \\ + \kappa_{\nu}(\mathcal{U}')P_{\nu}^{(2)}(\cos\theta_{q})L_{Ml}(\theta)]$$
(5)

with l'=l+1. In both Eqs. (1) and (5) the  $A_{\nu}(ll')$  can be expressed in terms of the relative populations of the substates of the emitting level

$$A_{\nu}(ll') = (-)^{l'-l} [(2l+1)(2l'+1)]^{1/2} C(ll'\nu; 1, -1)$$

$$\times \sum_{m_{i},m} (-)^{m+1} C(J_{i}lJ_{f}; m_{i}m) C(J_{i}l'J_{f}; m_{i}m)$$

$$C(ll'\nu; m, -m) P(m_{i}), \quad (6)$$

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where  $J_i$  and  $J_f$  are the spins of the initial and final states of the transition,  $m_i$  and  $m_f$  are the projections of  $J_i$  and  $J_f$  on the z axis,  $m = m_i - m_f$ , and  $\sum_{m_i} P(m_i)$ =1. In general,  $A_0(ll')=1$  and 0 for l'=l and  $l'\neq l$ , respectively. If the  $P(m_i)$  are equal, i.e., nonaligned nuclei, then  $A_{\nu}(ll')=0$  for  $\nu \neq 0$ .

The directional distribution of the accompanying gamma radiation is given by

$$W(\theta_{\gamma}) \sim \sum_{\nu} A_{\nu} P_{\nu}(\cos\theta_{\gamma}), \qquad (7)$$

where  $\theta_{\gamma}$  is the polar angle of the direction of emission of the gamma radiation and

$$A_{\nu} = [A_{\nu}(ll) + 2xA_{\nu}(ll') + x^{2}A_{\nu}(l'l')]/(1+x^{2}), \quad (8)$$

with l' = l+1.

For pure multipoles and nonaligned nuclei  $8\pi^2 F_l(\theta, \delta, \theta_q, \phi_q) = \gamma_l(\theta)$ . The general expression for the spectrometer efficiency is obtained by replacing  $\gamma_l(\theta)$ by  $8\pi^2 F_{M,E}(\theta, \frac{1}{2}\pi, \theta_q)$  in the expression obtained previously<sup>5</sup> for nonaligned nuclei, i.e.,

$$\eta_{M,E}(2\pi) = \frac{4\pi f(k)T^2 R(\frac{1}{4}k^2 - 1)}{\frac{1}{2}k} \int_0^{2\pi} F_{M,E}(\theta, \frac{1}{2}\pi, \theta_q) d\phi$$
$$= 4\pi f(k)T^2 R\eta_{M,E}'(2\pi). \quad (9)$$

T is the transmission of the spectrometer for monoenergetic electrons expressed as a fraction of a sphere, R is the momentum resolution  $\Delta p/p$  for pair lines and f(k) corrects for counting rate losses in the detecting system. The quantity  $f(k)T^2R$  is determined experimentally. The angle  $\phi$  is the difference between the azimuthal angles of  $\beta_+$  and  $\beta_-$  ( $\phi = \phi_+ - \phi_-$ ). For the pair spectrometer

$$\cos\theta_q = \cos\alpha/\dot{\cos}(\frac{1}{2}\theta); \tag{10}$$

$$\cos^2(\frac{1}{2}\theta) = 1 - \sin^2\alpha \, \sin^2(\frac{1}{2}\phi) \,,$$

where  $\alpha$  is a spectrometer constant (the polar angle of the  $\beta_+$  and  $\beta_-$  at emission) equal to  $45.7 \pm 1^\circ$ . Using Eq. (10)  $F_{M,E}(\theta, \frac{1}{2}\pi, \theta_q)$  can be expressed as a function of  $\phi$  only and  $\eta_{M,E'}(2\pi)$  evaluated from Eq. (9). The result can be written for pure multipoles in the form

$$\eta_l' = \mathcal{E}_l' \sum_{\nu} A_{\nu}(ll) \Delta_{\nu}^l(2\pi) , \qquad (11)$$

where  $\Delta_0^l(2\pi) = 1$ , and  $\mathcal{E}_l'$  is the efficiency factor for nonaligned nuclei given previously.<sup>5</sup> For a Ml, El+1mixture the efficiency is given in terms of

$$\eta_{M,E}'(2\pi) = \frac{\eta_l'(2\pi) + 2x\eta_{l,l+1}'(2\pi) + x^2\eta_{l+1}'(2\pi)}{1 + x^2}, \quad (12)$$

with

$$\eta_{l,l+1}'(2\pi) = \mathcal{E}_{l}' \sum_{\nu} A_{\nu}(l,l+1) \Delta_{\nu}^{l,l+1}(2\pi).$$
(13)

The  $\Delta_{\mu}^{ll'}(2\pi)$  can be expressed in the form

$$\Delta_{\nu}^{ll'}(2\pi) = \sum_{n=0}^{l+1} H_n^{(\nu)}(ll') I_n(2\pi) / \sum_{n=0}^{l+1} H_n^{(0)}(l) I_n(2\pi) , \quad (14)$$



FIG. 1. The alignment factor  $\Delta_2^{ll'}(2\pi)$  for  $\alpha = 45^{\circ}$  and transition energies from 1.022 to 9 MeV. The values for the high-energy limit are also shown.

[Eq. (13)] in a mixed Ml, El+1 transition (l'=l+1). For *El* radiation

$$\Delta_{\nu}{}^{l}(2\pi) = \sum_{n=0}^{l+1} G_{n}{}^{(\nu)}(l) I_{n}(2\pi) / \sum_{n=0}^{l+1} G_{n}{}^{(0)}(l) I_{n}(2\pi) \,. \tag{15}$$

The  $H_n^{(\nu)}(\mathcal{U}')$ ,  $G_n^{(\nu)}(l)$ , and  $I_n(2\pi)$  are functions of  $\alpha$ and k. The  $I_n(2\pi)$ ,  $G_n^{(0)} (\equiv G_n)$ , and  $H_n^{(0)} (\equiv H_n)$  have been given previously.<sup>5</sup>

We have calculated the  $\Delta_{\nu}^{l}(2\pi)$  for E1, M1, E2, M2, and E3 transitions with  $\nu_{max} = 4$  in the latter case, and the  $\Delta_{\nu}^{ll'}(2\pi)$  for mixed M1, E2 and M2, E3 transitions all for  $\alpha = 45^{\circ}$ . The results are shown in Figs. 1 and 2.

For pure multipole emission the  $A_{\nu}(ll)$  can, in principle, be obtained from the angular distribution of the accompanying gamma rays using Eq. (7), or from theoretical considerations. However, in the case of mixed transitions the fact that the spectrometer efficiency is multipole-sensitive while gamma-ray detectors are not means that the spectrometer efficiency cannot be evaluated simply from the gamma-ray angular distributions. To illustrate the evaluation of the spectrometer efficiency in the general case, we consider an M1, E2 mixture. Combining Eqs. (12) through (15) we have

$$\eta_{M,E'}(2\pi) = \frac{\mathcal{E}_{M1}' + x^2 \mathcal{E}_{E2}'}{1 + x^2} + \frac{x^2}{1 + x^2} \Delta_4^{E2}(2\pi) A_4(22) \mathcal{E}_{E2}' \\ \left[ \Delta_2^{M1}(2\pi) A_2(11) + 2x \Delta_2^{M1,E2}(2\pi) A_2(12) \right] \\ + \frac{x^2 (\mathcal{E}_{E2}' / \mathcal{E}_{M1}') \Delta_2^{E2}(2\pi) A_2(22) ] \mathcal{E}_{M1}'}{1 + x^2}.$$
(16)

Let us consider a specific case, namely the  $B^{10}$  3.58  $\rightarrow$ for *Ml* radiation (l'=l), or for the interference term 0.72 transition which has  $J_i=2^+$ ,  $J_f=1^+$ . For this,



FIG. 2. The alignment factor  $\Delta_4^{l\nu'}(2\pi)$  for  $\alpha = 45^{\circ}$  and transition energies from 1.022 to 9 MeV. The values for the high-energy limit are also shown.

transition the  $A_{\nu}$  are

$$A_{2} = [1 - 2P(0) - 3P(1)] \frac{0.5 + 2.236x - 0.3566x^{2}}{1 + x^{2}},$$

$$A_{4} = [1 + 5P(0) - 10P(1)] \frac{0.1904x^{2}}{1 + x^{2}}.$$
(17)

These can be derived from Eqs. (6) and (8), or more simply using the method of Litherland and Ferguson.<sup>33</sup> The angular distribution of a mixed gamma-ray transition relative to an arbitrary quantization axis can always be expressed in the form of Eq. (17); that is, the product of a function of the populations of the  $m_i$  substates which we designate  $F_{\nu}(J_i)$ , and a function of the mixing parameter x. The functions  $F_{\nu}(J_i)$ contain all of the information pertaining to the formation of the state and can sometimes be evaluated approximately or quite accurately, from theoretical considerations of the reaction mechanism. Obtaining the  $\Delta_{\nu}^{ll'}(2\pi)$  from Figs. 1 and 2, and the  $\mathcal{E}_{M1}$  and  $\mathcal{E}_{E2}$ from Figs. 2 and 3 of Wilkinson et al.,<sup>5</sup> the spectrometer efficiency for the 2.86-MeV  $B^{10}$  3.58  $\rightarrow$  0.72 transition becomes

$$\eta_{M,E}'(2\pi) \times 10^4 = \frac{0.395 + 0.470x^2}{1 + x^2} - 0.057F_4(2)\frac{x^2}{1 + x^2} + 0.395F_2(2)\frac{0.23 + 0.76x - 0.235x^2}{1 + x^2}.$$
 (18)

If the  $m_i$ -substates are populated equally then  $P(0) = P(1) = \frac{1}{5}$ , the  $F_{\nu}(J_i)$  are zero, and the efficiency reduces to that for a nonaligned nucleus. In the general case, full knowledge of x and the  $P(m_i)$  is necessary in

order to obtain an accurate value of the spectrometer efficiency. However, a knowledge of the form of the theoretical expression for the efficiency is quite important in itself since we can use it to estimate the uncertainty in the efficiency corresponding to any uncertainty in the values of x and the  $F_{\nu}(J_i)$ . Also, the dependence of the spectrometer efficiency on x can be used to obtain information on this parameter, at least in principle.

### III. RESULTS

# A. Be<sup>9</sup>+d Spectra at $E_d = 2.7$ and 2.0 MeV

### Procedure and Results

The internal pair-line spectra from the bombardment of a  $(3.7\pm0.3)$  mg/cm<sup>2</sup> thick Be foil target with 2.7and 2.0-MeV deuterons were observed with the full annulus opening (17 mm) of the spectrometer. The results are shown in Fig. 3. The expected resolution (full width at half-maximum) of the pair lines was derived from the 2.6% point-source instrumental linewidth, a contribution of 1% due to source diameter and a contribution of approximately 0.7% from Doppler broadening, or about 2.9% in all. The surface density of the target was measured by weighing to be 4.0



FIG. 3. Magnetic lens pair spectrometer results for Be<sup>9</sup>+d at  $E_d=2.7$  MeV (upper curve) and 2.0 MeV (lower curve). The pair lines are identified by the nucleus and the energy levels (in MeV) to which they are assigned. The resolution (full width at half-maximum) for these spectra is 2.95%.

<sup>&</sup>lt;sup>33</sup> A. E. Litherland and A. J. Ferguson, Can. J. Phys. **39**, 788 (1961).

	$E_{\gamma}$ (MeV)	Assignment	Doppler shiftª (keV)	$E_{\gamma}$ (corrected) (MeV)	Transition energy <sup>b</sup> (MeV)	Peak intensity (counts/µC)	Cross section° (mb)
-	$\begin{array}{c} 2.596 \pm 0.015 \\ 2.883 \pm 0.015 \\ 3.042 \pm 0.015 \\ 3.386 \pm 0.010 \\ 3.597 \pm 0.012 \\ 4.481 \pm 0.012 \\ 5.184 \pm 0.015 \\ 5.995 \pm 0.010 \end{array}$	$\begin{array}{c} {\rm Be^{10}} \; 5.96 \rightarrow 3.37 \\ {\rm B^{10}} \; 3.58 \rightarrow 0.72 \\ {\rm B^{10}} \; 5.16 \rightarrow 2.15 \\ {\rm Be^{10}} \; 3.37 \rightarrow 0 \\ {\rm B^{10}} \; 3.38 \rightarrow 0 \\ {\rm B^{10}} \; 5.16 \rightarrow 0.72 \\ {\rm B^{10}} \; 5.16 \rightarrow 0.72 \\ {\rm B^{10}} \; 5.16 \rightarrow 0 \\ {\rm Be^{10}} \; 5.96 \rightarrow 0 \end{array}$	$12\pm 111\pm 414\pm 312\pm 314\pm 520\pm 525\pm 630\pm 2$	$\begin{array}{c} 2.584 \pm 0.015\\ 2.872 \pm 0.015\\ 3.028 \pm 0.015\\ 3.374 \pm 0.010\\ 3.583 \pm 0.013\\ 4.461 \pm 0.013\\ 5.159 \pm 0.016\\ 5.965 \pm 0.010\\ \end{array}$	$\begin{array}{c} 2.591 {\pm} 0.009 \\ 2.866 {\pm} 0.005 \\ 3.012 {\pm} 0.008 \\ 3.368 {\pm} 0.009 \\ 3.583 {\pm} 0.005 \\ 4.448 {\pm} 0.007 \\ 5.165 {\pm} 0.007 \\ 5.959 {\pm} 0.009 \end{array}$	$\begin{array}{c} 2.12 {\pm} 0.07 \\ 3.85 {\pm} 0.09 \\ 0.78 {\pm} 0.05 \\ 9.05 {\pm} 0.02 \\ 1.30 {\pm} 0.07 \\ 0.61 {\pm} 0.03 \\ 0.12 {\pm} 0.01 \\ 4.98 {\pm} 0.05 \end{array}$	$\begin{array}{c} 16.9 \ \pm 0.9 \\ 33.2 \ \pm 1.9 \\ 5.56 \pm 3.8 \\ 51.3 \ \pm 3.8 \\ 7.70 \pm 0.6 \\ 2.48 \pm 0.2 \\ 0.47 \pm 0.05 \\ 15.5 \ \pm 0.7 \end{array}$

TABLE I. Magnetic lens pair spectrometer results for gamma rays from  $Be^9+d$  at  $E_d=2.7$  MeV.

• Experimental or estimated (see text). • Best values for the level separations from Ref. 11. • Average value for  $E_d = 2.0$  to 2.7 MeV. The absolute cross-section scale has an estimated uncertainty of 20% in addition to the relative errors which are given.

 $mg/cm^2$ . The uncertainty in the Be target thickness  $(3.7\pm0.3 \text{ mg/cm}^2)$  reflects an inexact knowledge of the carbon and oxygen impurities in the foil. Positron activities induced by deuteron bombardment of these impurities were responsible for a part of the background apparent in the spectra of Fig. 3.

The results for  $E_d = 2.7$  MeV are summarized in Table I. Measured energies of the transitions, given in the first column of Table I, are averages taken from the results on one complete spectrum for transition energies from 2.5 to 6.4 MeV (Fig. 3) and from several partial spectra, some taken at 1.8% resolution. The nuclear pair line from the  $0^+$  first excited state of  $O^{16}$ at<sup>11</sup>  $6.052 \pm 0.004$  MeV formed by means of the  $F^{19}(p,\alpha)O^{16}$  reaction provided the spectrometer energy calibration.

The internal pair lines were assigned to the transitions listed in the second column of Table I. All of these transitions have been observed previously by one means or another.<sup>11</sup>

In order to compare the measured energies with the expected transition energies a correction was applied for the Doppler shift due to the average forward velocity of the recoiling nuclei. Neglecting the possible correlation between the recoiling nuclei and the electron-positron pair, the fractional change in the transition energy  $E_{\gamma}$  of a pair line due to the Doppler shift can be shown to be

$$\Delta E_{\gamma}/E_{\gamma} = F'\beta_{\rm c.m.}\beta_{e}\cos\alpha [1+\gamma^{-1}\langle\cos\theta_{\rm c.m.}\rangle], \quad (19)$$

where F' is a fraction,<sup>27</sup>  $0 \leq F' \leq 1$ , which takes into account the average slowing down of the recoiling nuclei before the gamma rays are emitted,  $c\beta_{c.m.}$  is the velocity of the center of mass in the laboratory system,  $c\beta_e$  is the speed of the electron and positron (they have equal energies) in the center-of-mass system and is given by  $[1-(2/k)^2]^{1/2}$ ,  $\alpha$  is the average acceptance angle (=45.7°) of the spectrometer,  $\gamma$  is the ratio of the speed of the center of mass in the laboratory system to the speed of the recoiling nucleus in the center-of-mass system, and  $\langle \cos\theta_{c.m.} \rangle$  is the average over the angular distribution of the cosine of the angle of the recoiling nucleus to the z axis (i.e., the beam and spectrometer axis) in the center-of-mass system. Since  $\Delta E_{\gamma}/E_{\gamma}$  is a function of the bombarding particle energy it must be averaged over the target thickness.

Estimates of the Doppler shifts are given in the third column of Table I. The shifts of the Be<sup>10</sup> pair lines were evaluated from previous work<sup>27</sup> on the Doppler shifts of the corresponding gamma-ray transitions which essentially gave  $F'\beta_{\rm e.m.} [1+\gamma^{-1}\langle\cos\theta_{\rm e.m.}\rangle]$ . For the transitions from the B<sup>10</sup> 3.58- and 5.16-MeV levels, the Doppler shifts were calculated assuming an average deuteron energy of 2.4 MeV and an average center-of-mass angle of the neutrons in the  $Be^{9}(d,n)B^{10*}$ reaction of  $70^{\circ}$ . Both of these levels are expected to be formed by the stripping reaction with a transferred proton orbital angular momentum,  $l_p = 1$ . The value of  $\langle \cos\theta_{c.m.} \rangle$  assumed is consistent with other l=1deuteron stripping reactions for similar kinematical conditions. For the B10 pair lines the uncertainties on the Doppler shifts are mostly due to estimates of the probable deviation from the assumed value of  $\langle \cos\theta_{c.m.} \rangle$ . The effect of a correlation between the recoiling nuclei and the electron-positron pair was considered and it was found that the effect on the Doppler shifts is negligible compared to other errors. This can be illustrated best if the correlation is limited to the form  $1 + A_2 P_2(\cos\theta)$ ; that is, if there are no terms in  $P_4(\cos\theta)$ . Then, for the spectrometer mean acceptance angle of 45.7°,  $P_2(\cos\alpha)$  has a value of 0.25, which is relatively small, and the anisotropy correction factor which is proportional to a  $A_2P_2(\cos\theta)$  will also be relatively small.

The measured transition energies, corrected for the Doppler shifts, are given in the fourth column of Table I while the best values of the level separations<sup>11</sup> are given in the fifth column. A comparison of these two columns shows that the measured transition energies are all within one standard deviation of the expected transition energies.

The measured peak intensities in counts/ $\mu$ C are given in the sixth column of Table I. These peak intensities were converted to average total cross sections for the deuteron energy range from 2.0 to 2.7 MeV (corresponding to the measured target thickness) by

Assignment	Peak intensity $(counts/\mu C)$	Assumed spin-parity and multipolarity	Assumed $A_{\nu}$ (see text)	Cross section <sup>a</sup> (mb)
$Be^{10} 5.96 \rightarrow 3.37$	< 0.10	$1^- \rightarrow 2^+:E1$	$A_2 = A_4 = 0$	<0.8
$Be^{10} 3.58 \rightarrow 0.72$	$4.68 {\pm} 0.07$	$2^+ \rightarrow 1^+: M1, E2$	$A_2 = A_4 = +(0.014 \pm 0.04)$	$41.0 \pm 2.2$
$B^{10}$ 5.16 $\rightarrow$ 2.15	$0.67 \pm 0.04$	$2^+ \rightarrow 1^+: M1$	$A_2 = +0.21, A_4 = 0$	$4.79 \pm 0.36$
$\mathrm{Be^{10}}$ 3.37 $\rightarrow$ 0	$5.45 \pm 0.12$	$2^+ \rightarrow 0^+$ : E2	$A_2 = +(0.03 \pm 0.05)$ $A_4 = +(0.06 \pm 0.05)$	$30.8 \pm 2.4$
$B^{10}  3.58 \rightarrow 0$	$1.64{\pm}0.4$	$2^+ \rightarrow 3^+: M1, E2$	$A_2 = + (0.03 \pm 0.05)$ $A_4 = - (0.01 \pm 0.08)$	$9.80 {\pm} 0.56$
$B^{10}$ 5.16 $\rightarrow 0.72$	$0.54 \pm 0.02$	$2^+ \rightarrow 1^+: M1$	$A_{1} = +0.21$ , $A_{4} = 0$	$2.20 \pm 0.17$
$B^{10}$ 5.16 $\rightarrow$ 0	$0.10 \pm 0.01$	$2^+ \rightarrow 3^+: M1$	$A_{3} = +0.06, A_{4} = 0$	$0.39 \pm 0.05$
$Be^{10}$ 5.96 $\rightarrow$ 0	$0.05 \pm 0.02$	$1^- \rightarrow 0^+:E1$	$A_2 = A_4 = 0$	$0.15 \pm 0.06$

TABLE II. Magnetic lens pair spectrometer results for gamma rays from  $Be^9+d$  at  $E_d=2.0$  MeV.

<sup>a</sup> Average value for  $E_d = 1.0$  to 2.0 MeV. The absolute cross-section scale has an estimated uncertainty of  $\pm 20\%$  in addition to the relative errors which are given

means of the relationship,

$$\sigma = (\text{peak intensity})/nN\eta, \qquad (20)$$

where *n* is the incident flux of deuterons/ $\mu C$ , *N* the number of Be<sup>9</sup> nuclei/cm<sup>2</sup> and  $\eta$  the efficiency of the spectrometer. The efficiency is given by (Sec. II)

$$\eta = 4\pi f(k) T^2 R \eta'. \tag{21}$$

The factor f(k) was measured with an absolute accuracy of 5% and a relative accuracy of 2%, the transmission was measured to be  $0.080 \pm 0.005$  at a resolution R of 2.9% (annulus setting of 17 mm).

The results for  $E_d = 2.0$  MeV are summarized in Table II. For both the 2.0- and 2.7-MeV results the efficiency factors,  $\eta'$ , were evaluated from the results of the last section using the assumed spin-parities and multipolarities given in the third column of Table II and the anisotropy coefficients given in the fourth column.

For the transitions in Tables I and II assumed to be pure multipoles, the angular distributions of the gamma rays associated with a given transition can be used to determine the  $A_{\nu}(l)$  of Sec. II [see Eqs. (7) and (8)]. The origin of the  $A_{\nu}$ 's is given in the detailed discussion of the transitions given below. The last columns of Tables I and II give the cross sections evaluated from Eq. (20) using the  $A_{\nu}$  of Table II, the  $\Delta_{\nu}{}^{l}(2\pi)$  of Figs. 1 and 2, and the  $\mathcal{E}_i$  calculated<sup>5</sup> for nonaligned nuclei. The efficiency factor  $\mathcal{E}_{l}$  appropriate for nonaligned nuclei, was corrected for the effects of finite resolution and has an estimated uncertainty of 2%. The correction varied between 0.2 and 2% for transition energies between 2.5 and 6 MeV, respectively. Because the  $\Delta_{\nu}^{l}(2\pi)$  contribute a small correction to the efficiency, the effect of the uncertainty in the  $\Delta_{\mu}^{l}(2\pi)$  is negligible. The uncertainties given for the cross sections in the last column of Table I and II are due mainly to the statistical uncertainties in the peak intensities and our estimates of the possible effects of uncertainties in the Α,.

Since the efficiency of the spectrometer is a function of the multipolarity of the transition, the cross section of a given transition would be changed if the assumed multipolarity were wrong. However, for all the transitions but those from the  $B^{10}$  3.58-MeV level the multipolarity assignments seem quite definite.<sup>11</sup> For the  $3.58 \rightarrow 0.72$  and  $3.58 \rightarrow 0$  transitions strong M1-E2 mixtures are possible.<sup>24,34</sup> The possible effects of M1-E2mixing on these two transitions are discussed below. The cross section scales of Tables I and II have an estimated uncertainty of 20% in addition to the relative errors assigned to the individual transitions.

The procedure used to obtain the cross sections was checked by measuring the thick target yield of the 3.56-MeV transition from the Be<sup>9</sup> $(p,\alpha\gamma)$ Li<sup>6</sup> resonance at  $E_p = 2.56$  MeV. The yield of this isotropic transition was found to be  $5.0 \times 10^{-6}$  gamma-rays/proton at  $E_p = 2.72$  MeV in good agreement with the best previous value<sup>35</sup> of  $4.7 \times 10^{-6}$  gamma-rays/proton which has an estimated accuracy of 10-15%.

# Discussion

1. The B<sup>10</sup> 4.77-MeV level. In previous work<sup>11</sup> on the gamma rays from  $Be^9 + d$  which have energies greater than 2.5 MeV a transition from the B<sup>10</sup> 4.77-MeV level was reported in addition to those identified in Tables I and II. In the present work no evidence was seen for transitions from the B<sup>10</sup> 4.77-MeV level. Investigation<sup>18,36</sup> of the Li<sup>6</sup> $(\alpha, \gamma)$ B<sup>10</sup> reaction shows that this level decays  $\sim 100\%$  to the B<sup>10</sup> 0.72-MeV level with a possible weak ground-state branch. From the data of Fig. 3 upper limits on the cross sections of these two transitions of 0.5 and 0.3 mb, respectively, at both deuteron energies can be fixed. The previous evidence<sup>37</sup> for the  $B^{10}$  4.77  $\rightarrow$  0.72 transition following the  $Be^{9}(d,n\gamma)B^{10}$  reaction would indicate a cross section for the  $Be^{9}(d,n)B^{10}$  (4.77  $\rightarrow$  0.72) transition about five times the present limit given; however, this evidence was not definite and it is concluded that transitions from the B<sup>10</sup> 4.77-MeV level have not been observed in the  $Be^{9}(d,n\gamma)$  reaction.

<sup>&</sup>lt;sup>34</sup> S. M. Shafroth and S. S. Hanna, Phys. Rev. 104, 399 (1956).
<sup>35</sup> R. B. Day and R. L. Walker, Phys. Rev. 85, 582 (1952).
<sup>36</sup> H. Warhanek, Phil. Mag. 2, 1085 (1957).
<sup>37</sup> W. E. Meyerhof and L. F. Chase, Jr., Phys. Rev. 111, 1348 (1958); and private communication.

An upper limit to the partial gamma-ray width,  $\Gamma_{\gamma}/\Gamma$  of the B<sup>10</sup> 4.77-MeV level can be set from the upper limit to the  $Be^{9}(d,n)B^{10}$  (4.77-MeV level) cross section and previous work on the same reaction. From the data of Ajzenberg,38 it appears that the B10 4.77-MeV level is formed in the  $Be(d^9,n)B^{10}$  reaction at an average deuteron energy of 3.39 MeV with a cross section within a factor of two of that for the B<sup>10</sup> 3.58-MeV level. From Tables I and II it is seen that the cross sections at  $E_d = 2.7$  and 2.0 MeV for formation of the B<sup>10</sup> 3.58-MeV level are both greater than 40 mb (the B<sup>10</sup> 3.58-MeV level decay is known to have a  $\sim 20\%$  branch to the B<sup>10</sup> 2.15-MeV).<sup>11</sup> Assuming that the relative cross sections for the formation of the  $B^{10}$ 3.58- and 4.77-MeV levels are approximately the same at 2.0, 2.7, and 3.39 MeV, we find that  $\Gamma_{\gamma}/\Gamma \leq 0.5/40$ for the B<sup>10</sup> 4.77-MeV level. We raise this limit by a factor of four to allow for the assumptions made and conclude that  $\Gamma_{\gamma}/\Gamma < 0.05$  for the B<sup>10</sup> 4.77-MeV level.

2. The  $B^{10}$  levels. No evidence was obtained for the Be<sup>10</sup> 6.18  $\rightarrow$  0, 6.18  $\rightarrow$  3.37 or 6.26  $\rightarrow$  0 transitions for the  $E_d = 2.7$ - and 2.0-MeV data. Transitions from the Be<sup>10</sup> 6.18-MeV level have not been reported. There is no evidence on the spin-parity assignment of this level. A Be<sup>10</sup>  $6.26 \rightarrow 0$  transition has not been observed and it has been suggested<sup>35</sup> but not definitely proven that the Be<sup>10</sup> 6.26-MeV level is  $2^-$  and decays by an E1 cascade through the Be<sup>10</sup> 3.37-MeV level. The Be<sup>10</sup>  $6.26 \rightarrow 3.37$  transition has an expected<sup>11</sup> energy of 2.894 MeV as compared to 2.866 MeV for the B10  $3.58 \rightarrow 0.72$  transition and thus these two transitions would be unresolved in the 2.7-MeV spectrum of Fig. 3. The threshold for the  $Be^{9}(d,p)Be^{10}$  (6.26-MeV level) reaction is 2.05 MeV and thus the Be<sup>10</sup> 6.26-MeV level cannot be formed at  $E_d = 2.0$  MeV. The contribution of the Be<sup>10</sup>  $6.26 \rightarrow 3.37$  transition to the 2.87-MeV peak of Fig. 3 (and Table I) at  $E_d = 2.7$  MeV can be obtained by comparing the relative intensities of the peaks corresponding to the  ${\rm B^{10}}\ 3.58 {\,\rightarrow\,} 0$  and  $3.58 {\,\rightarrow\,} 0.72$ transitions at  $E_d = 2.7$  and 2.0 MeV. From the 2.0-MeV data (Table II) we obtain a ratio of  $4.20\pm0.3$  for the intensity of the  $\rm B^{10}$   $3.58 \rightarrow 0.72$  transition relative to the B<sup>10</sup>  $3.58 \rightarrow 0$  transition, while the 2.7-MeV data (Table I) yields  $4.33 \pm 0.4$  for this ratio. From these ratios we deduce that the average cross section for the  $\operatorname{Be}^{9}(d,p)\operatorname{Be}^{10}$  (6.26  $\rightarrow$  3.37) reaction for the interval  $E_d = 2.0-2.7$  MeV is  $1\pm 3$  mb if the transition is E1 as assumed. The low yield of the  $Be^{9}(d,p)Be^{10}$  (6.26-MeV level) reaction is presumably due to the fact that the yield of this endothermic reaction is still increasing rapidly from threshold ( $E_d = 2.05$  MeV) at  $E_d = 2.7$ MeV. In order to obtain higher relative yields for the transitions from the Be<sup>10</sup> 6.18- and 6.26-MeV levels, these transitions were investigated at  $E_d = 3.2$  MeV. The results will be presented in Sec. IIB.

The branching ratios of the gamma-ray decay of the

Be<sup>10</sup> 5.96-MeV level can be obtained from the 2.7-MeV data. From the relative cross sections of Table I, we obtain  $(48\pm2)\%$  and  $(52\pm2)\%$  for the Be<sup>10</sup> 5.96 $\rightarrow$ 0 and  $5.96\rightarrow3.37$  branches, respectively. For both transitions we assume  $A_2=A_4=0$  with negligible uncertainty since the Be<sup>10</sup> 5.96-MeV level is known<sup>11</sup> to be formed by the stripping reaction with  $l_n=0$  and thus the decay products of the level will be emitted isotropically with respect to the z axis.

The only previous measurement<sup>37</sup> of the branching ratio of the Be<sup>10</sup> 5.96-MeV level gave  $(78\pm 12)\%$  and  $(22\pm6)\%$  for the intensities of the 5.96  $\rightarrow$  3.37 and  $5.96 \rightarrow 0$  transitions, in rather poor agreement with the present results. The branching ratio reported by Meyerhof and Chase<sup>37</sup> was obtained using the  $\operatorname{Be}^{9}(d, p\gamma)\operatorname{Be}^{10}$  reaction and a three-crystal pair spectrometer which was well shielded with lead. In recent work at this laboratory<sup>39</sup> a strong 2.62-MeV gamma ray from  $Pb^{208}(n,n'\gamma)Pb^{208}$  was observed in three-crystal pair spectra (taken with lead shielding) when the neutron yield from the target was strong and of sufficiently high energy. The largest yield of 2.62-MeV gamma rays was observed for  $Be^9+d$ . Since the  $Be^{10}$  $5.96 \rightarrow 3.37$  transition has an energy of 2.59 MeV it seems likely that the large ratio for  $5.96 \rightarrow 3.37$  to  $5.96 \rightarrow 0$  reported by Meyerhof and Chase is due to a contribution from the Pb<sup>208</sup> 2.62-MeV gamma ray.

3. The  $Be^{1}$  3.37<sup>5</sup>MeV and  $B^{1}$  3.5<sup>2</sup>/<sub>3</sub>-MeV levels. In order to obtain some information on the  $A_{\nu}$  appropriate for the Be<sup>10</sup>  $3.37 \rightarrow 0$ , B<sup>10</sup>  $3.58 \rightarrow 0$  and B<sup>10</sup>  $3.58 \rightarrow 0.72$  transitions the gamma-ray spectra from Be<sup>9</sup>+d were recorded every 15 deg between 0° and 90° to the beam at a deuteron energy of 2.0 MeV. The Be<sup>9</sup> target was the same one used in the pair spectrometer work and the spectra were taken with a 5 in. $\times$ 5 in. NaI crystal with its front face 6.2 in. from the target. From these data the angular distributions of the Be<sup>10</sup>  $3.37 \rightarrow 0$  and  $B^{10}$   $3.58 \rightarrow 0.72$  transitions were measured. The  $3.58 \rightarrow 0$  transition was quite weak relative to the background and to the  $3.37 \rightarrow 0$  transition. For this reason its angular distribution was obtained relative to that of the  $B^{10}$  3.58  $\rightarrow$  0.72 transition from three-crystal pair spectra taken at  $0^{\circ}$ ,  $55^{\circ}$ , and  $75^{\circ}$  to the deuteron beam.<sup>39</sup>

The  $A_{\nu}$  given in Table II for the Be<sup>10</sup>  $3.37 \rightarrow 0$ transition are the result of a least-squares analysis of the data for that transition. The gamma-ray angular distributions were not studied at 2.7 MeV. For lack of further information the same  $A_{\nu}$  were assumed for the Be<sup>10</sup>  $3.37 \rightarrow 0$  transitions for the 2.7-MeV data as for the 2.0-MeV data. It does not seem likely that the  $A_{\nu}$  for the 2.7-MeV data have significantly larger magnitudes than for the 2.0-MeV data since about  $\frac{1}{3}$ of the intensity of the 3.37-MeV transition at 2.7 MeV is due to the Be<sup>10</sup>  $5.96 \rightarrow 3.37 \rightarrow 0$  cascade and this

<sup>&</sup>lt;sup>38</sup> F. Ajzenberg, Phys. Rev. 82, 43 (1951); 88, 298 (1952).

<sup>&</sup>lt;sup>39</sup> D. J. Bredin, J. W. Olness, and E. K. Warburton (to be published).

component should be isotropic. Since the  $A_2$  and  $A_4$  given in Table II for the Be<sup>10</sup>  $3.37 \rightarrow 0$  transition have the same sign while  $\Delta_2^{E2}(2\pi)$  and  $\Delta_4^{E2}(2\pi)$  have about the same magnitude but opposite signs (Figs. 1 and 2), the alignment correction to the cross section is only -2% with an assumed uncertainty of 6%.

The least-squares analysis of the data for the B<sup>10</sup>  $3.58 \rightarrow 0.72$  transition yielded  $A_2 = A_4 = 0.014 \pm 0.04$ . The measurements on the  $3.58 \rightarrow 0$  transitions yielded  $A_2 = + (0.03 \pm 0.05)$  and  $A_4 = - (0.01 \pm 0.08)$ . Thus, both angular distributions are isotropic within the uncertainty of the measurements.

The decay of the B<sup>10</sup> 3.58-MeV level formed in the  $Be^{9}(d,n)B^{10}$  reaction has been studied previously by Shafroth and Hanna<sup>34</sup> and by Garg et al.<sup>17</sup> Shafroth and Hanna studied the B<sup>10</sup>  $3.58 \rightarrow 0.72 \rightarrow 0$  correlation using a thick Be<sup>9</sup> target and deuteron energies less than 0.7 MeV. Their results were consistent with equal population of the  $m_i$  substates and with this condition yielded  $0.12 \leq x \leq 0.45$ ,  $x \geq +7.5$ , and  $x \leq -6$  as the possible values of the E2, M1 mixing parameter for the  $3.58 \rightarrow 0.72$  transition. Garg et al.<sup>17</sup> studied the  $(n,\gamma)$ correlation in the Be<sup>9</sup>( $d,n\gamma$ )B<sup>10</sup> reaction at  $E_d = 3.3$  and 4.4 MeV. At both deuteron energies they found the correlation between the neutrons detected at  $25^{\circ}$  to the deuteron beam and the gamma rays emitted by the 3.58-MeV level was isotropic to 5%. In this experiment they used a gamma-ray counter with a bias of about 2.3 MeV. Since the  $3.58 \rightarrow 0.72$  transition is about 4 times as strong as the  $3.58 \rightarrow 0$  transition they were measuring, to first approximation, the correlation of neutrons with the  $3.58 \rightarrow 0.72$  transition. The isotropy of the correlation measured by Garg et al. and of the distribution measured in the present work demands that  $A_2 \approx A_4 \approx 0$  in both experiments. The vanishing of  $A_4$  is not surprising since the Be<sup>9</sup>(d,n)B<sup>10</sup> reaction to the  $B^{10}$  3.58-MeV level is predominantly a stripping reaction with  $l_p = 1$  at the energies considered, and for this reaction  $A_4$  must be zero, i.e.,  $F_4(2) = 1$ +5P(0)-10P(1)=0. This restricts  $F_2(2)=1-2P(0)$ -3P(1) to values between 0.7 (corresponding to all S=2 in the channel spin formalism) and -0.7 (corresponding to all S=1). The vanishing of  $A_2$  can be due to  $F_2(2) = 1 - 2P(0) - 3P(1) = 0$  (equal mixtures of S = 1and S=2) or to x=-0.22 or x=+6.5, that is, to the vanishing of the function of x in Eq. (17). The value x = -0.22 is not consistent with the range  $0.12 \le x \le 0.45$ allowed by the results of Shafroth and Hanna<sup>34</sup>; thus, if the  $3.58 \rightarrow 0.72$  transition is predominantly M1 the 3.58-MeV level is formed with the  $P(m_i)$  equal, or nearly so, for all the experimental conditions considered here. If the  $B^{10}$  3.58  $\rightarrow$  0.72 transition is predominantly E2 it seems most likely that  $F_2(2) = 1 - 2P(0) - 3P(1)$ is close to zero but the possibility exists that  $A_2$  vanishes because x = +6.5 which is close to agreement with the condition  $x \ge 7.5$  from the measurements of Shafroth

and Hanna.<sup>34</sup> The  $A_{\nu}$  for the B<sup>10</sup> 3.58  $\rightarrow$  0 transition are

$$A_{2} = [1 - 2P(0) - 3P(1)] \frac{0.143 - 1.57x + 0.818x^{2}}{1 + x^{2}},$$

$$A_{4} = [1 + 5P(0) - 10P(1)] \frac{0.0204x^{2}}{1 + x^{2}}.$$
(22)

Since the  $F_{\nu}(2)$  are the same for any transition from the 3.58-MeV level [compare Eqs. (17) and (22)], the efficiency for the  $3.58 \rightarrow 0$  transition, as well as for the  $3.58 \rightarrow 0.72$  transition, will contain no interference terms if the  $F_{\nu}(2)$  are negligibly small. We note that the small measured values of the  $A_{\nu}$  for the  $3.58 \rightarrow 0$ transition strengthens the probability that  $F_{\nu}(2)$  is close to zero.

The cross sections given in Tables I and II for the  $B^{10} 3.58 \rightarrow 0$  and  $3.58 \rightarrow 0.72$  transitions were evaluated assuming that both transitions have  $0 \leq |x| \leq 0.45$  with negligible interference, i.e., the  $F_{\nu}(2)=0$ . Because the  $\mathcal{E}_{M1}'$  and  $\mathcal{E}_{E2}'$  differ by less than 20%, the possibility of an E2 mixture with  $0 \leq x^2 \leq 0.2$  introduces only a 3% uncertainty in the cross sections if the interference terms are negligible.

If the B<sup>10</sup>  $3.58 \rightarrow 0.72$  and  $3.58 \rightarrow 0$  transitions are both predominantly M1 the relative intensities of these two branches are  $(4.2\pm0.3)$ :1. If the interference terms are negligible then the relative intensities are 4.1:1, 3.6:1, and 4.8:1 for both transitions E2, the  $3.58 \rightarrow$ 0.72 E2 and the  $3.58 \rightarrow 0$  M1, and the  $3.58 \rightarrow 0.71$  M1and the  $3.58 \rightarrow 0$  E2, respectively. We adopt 4.2:1 for purposes of later discussion but keep in mind the possible deviations from this value.

4. The B<sup>10</sup> 5.16-MeV level. Because of the strength of the decays from the B<sup>10</sup> 5.16-MeV level it is unlikely that any of the three branches observed contain significant E2 contributions. This point is touched on in Sec. I and again in Sec. IV. Assuming that all three branches are pure M1 we have  $A_2=0.5F_2(2)$  [from Eq. (17)] for the  $5.16 \rightarrow 2.15$  and  $5.16 \rightarrow 0.72$  transitions and  $A_2=0.143F_2(2)$  [from Eq. (22)] for the  $5.16 \rightarrow 0$  transition, where  $F_2(2)=1-2P(0)-3P(1)$ . The function  $F_2(2)$  can be evaluated theoretically.

Garg et  $al^{17}$  measured the  $(n,\gamma)$  angular correlation combination of the B<sup>10</sup> 5.16  $\rightarrow$  2.15 and 5.16  $\rightarrow$  0.72 transitions. Both transitions have the same angular correlation for our assumptions of pure M1 decays. These measurements were made at deuteron energies between 2.65 and 3.3 MeV. If we average the  $A_2$ coefficients they obtained from these measurements we obtain +0.33. The  $(p,\gamma)$  angular correlation in the Be<sup>9</sup>(d,p)Be<sup>10</sup> reaction to the Be<sup>10</sup> 3.37-MeV level has also been measured.<sup>40</sup> In this work it was found that the angular correlation could be fitted by plane-wave stripping theory with the Be<sup>10</sup> 3.37-MeV level formed by 90% S=2, and 10% S=1, where S is the channel

<sup>&</sup>lt;sup>40</sup> S. A. Cox and R. M. Williamson, Phys. Rev. 105, 1799 (1957).

spin. Since the Be<sup>10</sup> 3.37-MeV level and the B<sup>10</sup> 5.16-MeV level are analog states, plane-wave stripping theory would demand that the channel spin ratio should be the same in the formation of the  $B^{10}$  5.16-MeV level by the  $Be^{9}(d,n)B^{10}$  reaction. The plane-wave stripping prediction for  $A_2$  with 90% S=2 and 10% S=1 is  $A_2=0.28$  for a  $2^+ \rightarrow 1^+$  transition following capture of  $l_p = 1$  protons. This value is in good agreement with the results of Garg et al.<sup>17</sup> The channel-spin ratio, 90% S=2, 10% S=1, corresponds to  $F_2(2)$ =1-2P(0)-3P(1)=0.56. Using the method of Warburton and Chase<sup>41,12</sup> which is applicable to endothermic deuteron stripping reactions near threshold we find that the plane-wave stripping prediction for the angular distributions of the  $5.16 \rightarrow 2.15$  and  $5.16 \rightarrow 0$  transitions are  $A_2 = 0.28Q_2$  and  $0.08Q_2$ , respectively, where  $Q_2$  is an attenuation factor,  $0 \leq Q_2 \leq 1$ , given to a good approximation in the present case by  $Q_2 = 1 - (k_n/k_d)^2$ where  $k_n$  and  $k_d$  are the wave numbers of the incident deuteron and outgoing neutron in the center-of-mass system. For the 2.7-MeV data the average value of  $Q_2$ (evaluated at  $E_d = 2.4$  MeV) is 0.7, while for the 2.0-MeV data the average value of  $Q_2$  (evaluated at  $E_d = 1.5$ MeV) is 0.8. We take an average and assume  $Q_2$  is 0.75 which results in  $A_2 = 0.21$  for the B<sup>10</sup> 5.16  $\rightarrow$  2.15 and 5.16  $\rightarrow$  0.72 transitions and  $A_2 = 0.06$  for the 5.16  $\rightarrow$  0 transition. Using these values and  $\Delta_2^{M1}(2\pi)$  from Fig. 1, the correction for alignment raises the cross sections of the 5.16  $\rightarrow$  2.15 and 5.16  $\rightarrow$  0.72 transitions by 10% and that for the  $5.16 \rightarrow 0$  transition by 2.8%. An additional 5% uncertainty in the cross section was assumed due to the uncertainty in this alignment correction. The angular distribution of the  $B^{10}$  5.16  $\rightarrow$ 0.72 gamma-ray transition was also obtained from the three-crystal pair spectra.<sup>39</sup> The result is  $A_2 = 0.27 \pm 0.1$ in agreement with the predicted value of 0.21.42

If we assume that the decay of the B<sup>10</sup> 5.16-MeV level is negligible to B<sup>10</sup> states between 2.15 and 5.16 MeV, the branching ratios of this state to the levels at 2.15, 0.72, and 0 MeV can be obtained from both the  $E_d$ =2.7 and 2.0 MeV data. From Tables I and II the results are found to be (65.4±3)%, (29.2±3)%, and (5.5±0.8)% from the 2.7-MeV data and (65.0 ±3)%, (29.8±3)%, and (5.3±0.8)% from the 2.0-MeV data. The two results are in agreement and we adopt (65±2)%, (29.5±2)%, and (5.5±0.7)% for the branching ratios of the B<sup>10</sup> 5.16-MeV level to the B<sup>10</sup> levels at 2.15, 0.72, and 0 MeV, respectively. The branching ratios of the B<sup>10</sup> 5.16-MeV level have been determined previously by means of the Be<sup>9</sup>(d,n)B<sup>10</sup> reaction and the Li<sup>6</sup>( $\alpha,\gamma$ )B<sup>10</sup> reaction. For the former results various investigations<sup>11</sup> combine to give<sup>12</sup> (52  $\pm 22$ )%, (38 $\pm 17$ )%, and (10 $\pm 6$ )% for the 5.16 $\rightarrow$ 2.15, 5.16 $\rightarrow 0.72$ , and 5.16 $\rightarrow 0$  transitions, respectively; while the latter gives<sup>18</sup> 64%, 29%, and 7%, respectively, with estimated uncertainties of about 25%. The present results are in good agreement with these previous determinations.

Assuming that the B<sup>10</sup> 5.16-MeV level and the Be<sup>10</sup> 3.37-MeV level are two of the three  $J^{\pi}=2^+$ , T=1members of a mass 10 isotopic-spin triplet (as is expected) the partial gamma-ray width  $\Gamma_{\gamma}/\Gamma$ , of the B<sup>10</sup> 5.16-MeV level can be estimated from the relative cross sections of the  $Be^{9}(d,n)B^{10*}$  (5.16  $\rightarrow$  0.72) and  $\operatorname{Be}^{9}(d,p)\operatorname{Be}^{10*}(3.37 \rightarrow 0)$  reactions. If we were to assume the simplest direct interaction mechanism (plane-wave stripping theory) or compound-nucleus formation and, in addition, neglected kinematical and penetrability factors, the cross section for forming the 3.37-MeV level should be twice that for forming the 5.16-MeV level since the neutron reduced width of the 3.37-MeV level is predicted to be twice the proton reduced width of the B<sup>10</sup> 5.16-MeV level if isotopic spin is a good quantum number. Then the partial gamma-ray width of the B<sup>10</sup> 5.16-MeV level would be given by

$$\Gamma_{\gamma}/\Gamma \simeq \frac{2}{0.29} \frac{\sigma(5.16 \to 0.72)}{\sigma(3.37 \to 0)}, \qquad (23)$$

which gives 0.54 for both the 2.7- and 2.0-MeV data. A rough excitation curve for the  $Be^{9}(d,n)B^{10}$  (5.16  $\rightarrow$ 0.72) transition was obtained from  $E_d = 1.2$  to 3.0 MeV with a  $1 \text{ mg/cm}^2$  Be<sup>9</sup> target and it was found that the yield was smoothly varying and constant within the error of the measurement  $(\pm \sim 20\%)$  for this energy range. A yield curve had been obtained previously<sup>10</sup> for the  $Be^{9}(d,p)Be^{10}$  (3.37  $\rightarrow$  0) transition, and it also showed little structure between deuteron energies of 1.0 and 2.7 MeV. The excellent agreement of the values of  $\Gamma_{\gamma}/\Gamma$  obtained from the 2.7- and 2.0-MeV data and the absence of pronounced structure in the excitation curves for the B<sup>10</sup> 5.16 $\rightarrow$ 0.72 and Be<sup>10</sup> 3.37 $\rightarrow$ 0 transitions may give some support to the present analysis. This value  $\Gamma_{\gamma}/\Gamma \approx 0.5$  is in agreement with the only previous determination<sup>12</sup>:  $\Gamma_{\gamma}/\Gamma = 0.7 \pm 0.35$ .

This analysis is, however, unacceptably oversimplified. The penetrability and kinematic factors are not large, and tend to cancel, and it is known that both the (d,p) and (d,n) reactions show strong direct interaction characteristics for the present levels even at low deuteron bombarding energies. But since our energies are low, both for the bombarding deuterons and for the outgoing nucleons, the effects of distorted waves might be expected to be severe. These have been estimated for a variety of distorting potentials and, unfortunately, the predicted (d,p) to (d,n) cross-section ratios vary considerably over the reasonable range of parameters.

<sup>&</sup>lt;sup>41</sup> E. K. Warburton and L. F. Chase, Jr., Phys. Rev. **120**, 2095 (1960).

<sup>&</sup>lt;sup>42</sup> To avoid possible confusion we emphasize that Eqs. (17) and (22) are valid for any quantization axis; but the  $P(m_i)$  depend on the axis chosen. Thus, in the  $(n,\gamma)$  correlation results the quantization axis is the recoil direction while for gamma-ray angular distributions it is the beam direction. The  $Q_r$  represent the attenuation of the  $F_r(J_i)$  when we average the  $(d,n\gamma)$  reaction over the possible recoil directions.



FIG. 4. Magnetic lens pair spectrometer results for Be<sup>9</sup>+d at  $E_d = 3.2$  MeV. The expected positions of pair lines due to transitions of given energies (in MeV) are indicated by the arrows. All transitions are in Be<sup>10</sup> except for the B<sup>10</sup> 2.866-MeV transition. The arrow marked D.S. indicates the expected position of the Be<sup>10</sup> 6.178-MeV pair line if it had a Doppler shift corresponding to a lifetime  $\ll 5 \times 10^{-13}$  sec.

For example, if we choose a reasonable set of nuclear parameters ( $V_n=41$ ,  $V_p=55$ ,  $W_n=0$ ,  $W_p=5$  MeV;  $r_{0n}=r_{0p}=1.3$ ,  $a_n=a_p=0.65$  F for a central Saxon-Woods potential in the usual notation) and vary  $V_d$ from 25 to 45 MeV, holding constant  $W_d=6$  MeV;  $r_{0d}=1.5$ ,  $a_d=0.6$  F, the (d,n) to (d,p) cross-section ratio predicted varies by a factor of more than 2. The result is not so sensitive to changes in the other parameters but this considerable sensitivity to  $V_d$  means that we cannot trust such computations for our present purpose and must state  $\Gamma_{\gamma}/\Gamma$  in terms of the as-yet-unmeasured cross-section ratio  $R=\sigma_{(d,p)}/\sigma_{(d,n)}$  for production of the states in question (suitably averaged to the conditions of our experiment):

$$\Gamma_{\gamma}/\Gamma = (R/0.29)\sigma(5.16 \to 0.72)/\sigma(3.37 \to 0).$$
 (24)

Since the values of R given by the distorted-wave parameters listed above and other reasonable sets are smaller than the zero-order value of 2 by factors as large as 4 it appears that  $\Gamma_{\gamma}/\Gamma$  may be rather small and this at least gives us a little confidence in deriving absolute values of  $\Gamma_{\gamma}$  from the measured  $(\alpha, \gamma)$  cross sections by assuming that the alpha-particle width is dominant.

# B. Transitions from the Be<sup>10</sup> 6.18- and 6.26-MeV Levels at $E_d = 3.2$ MeV

## The Be<sup>10</sup> 6.26-MeV Level

In order to study the decay modes of the Be<sup>10</sup> 6.18and 6.26-MeV levels the regions of transition energy between 2.7 and 3.0 MeV and between 5.9 and 6.3 MeV were studied at a deuteron energy of 3.2 MeV and with a spectrometer resolution of 1.3% using the  $(3.7\pm0.3)$ mg/cm<sup>2</sup> Be target. The results are shown in Fig. 4. It is clear from this figure that the resolution used was not sufficient completely to resolve the pair lines from the Be<sup>10</sup>  $6.26 \rightarrow 3.37$  and B<sup>10</sup>  $3.58 \rightarrow 0.72$  transitions but was sufficient to show that both lines exist, which had not been done heretofore, and to obtain an adequate measure of their energies and intensities.

The separation in energy of these two pair lines is measured from these data to be  $39\pm8$  keV. Assuming that the Doppler shifts of these two lines are the same, the energy separation of the two lines is calculated from the published<sup>11</sup> energy positions of the levels involved to be  $28\pm10$  keV, the sense of the slight disagreement is such as to suggest that, if anything, the Doppler shift of the B<sup>10</sup>  $3.58 \rightarrow 0.72$  transition is less than that of the Be<sup>10</sup>  $6.26 \rightarrow 3.37$  transition. The predicted energy separation of  $28\pm10$  keV is obtained from the energy difference of the B<sup>10</sup> 3.58- and 0.72-MeV levels,<sup>11</sup> and from the difference in energy of the Be<sup>10</sup> 6.26- and 3.37-MeV levels obtained by means of the Be<sup>9</sup>(d, p)Be<sup>10</sup> reaction.<sup>43</sup> For the latter we estimate an uncertainty of 9 keV.

No evidence was found for the Be<sup>10</sup>  $6.26 \rightarrow 0$  transition. Assuming that the Be<sup>10</sup> 6.26-MeV level is  $J^{\pi} = 2^{-}$ , which is the most probable value,<sup>11</sup> and that the  $6.26 \rightarrow 3.37$  and  $6.26 \rightarrow 0$  transitions are E1 and M2, respectively, an upper limit can be set on the relative strength of the  $6.26 \rightarrow 0$  branch from the data of Fig. 3. The result is that the Be<sup>10</sup>  $6.26 \rightarrow 0$  transition has an intensity less than 0.4% of the  $6.26 \rightarrow 3.37$  transition.

# The Be<sup>10</sup> 6.18-MeV Level

A ground-state transition from the Be<sup>10</sup> 6.18-MeV level was observed while a  $6.18 \rightarrow 3.37$  transition was not. Because of the background in the region of the expected position of the  $6.18 \rightarrow 3.37$  pair line and the weakness of the  $6.18 \rightarrow 0$  pair line, a sharp limit cannot be given for the relative intensity of the  $6.18 \rightarrow$ 3.37 transition; however, a useful limit can be obtained. If it were assumed that the  $6.18 \rightarrow 0$  pair transition is not E0 (i.e., the Be<sup>10</sup> 6.18-MeV level is not  $J^{\pi}=0^+$ ), the intensity of the  $6.18 \rightarrow 3.37$  transition would be less than 2.5 times that of the  $6.18 \rightarrow 0$  transition. This limit gives some information concerning the properties of the 6.18-MeV level. It cannot be  $J^{\pi}=0^{-1}$ since a  $0^- \rightarrow 0^+$  transition is too highly forbidden to be seen by the present means. An assignment of  $2^+$  is not very likely and assignments of  $2^-$  or  $J \ge 3$  are quite unlikely since these spin-parity assignments would most likely favor a transition to the 2+ 3.37-MeV level over one to the 0<sup>+</sup> ground state to a higher degree than observed. Thus, the limit on the relative intensities of the  $6.18 \rightarrow 3.37$  and  $6.18 \rightarrow 0$  transitions indicates that the most probable spin-parity assignment for the Be<sup>10</sup> 6.18-MeV level is  $J^{\pi} = 0^+$  or J = 1.

From the data shown in Fig. 4 for the  $Be^{10}$  5.96- and 6.18-MeV pair lines and a similar spectrum taken with

<sup>48</sup> J. J. Jung and C. K. Bockelman, Phys. Rev. 96, 1353 (1954).

1.8% resolution the difference in energy of the two pair lines was measured to be  $183\pm6$  keV. Jung and Bockelman<sup>43</sup> obtained a value of 219 keV for the energy separation of the Be<sup>10</sup> 5.96- and 6.18-MeV levels; from their published results and, guided by the stated errors in other work of the MIT group, we estimate  $\pm 5 \text{ keV}$ for the uncertainty in this measurement. The disparity between the present transition energy difference of  $183\pm6$  keV and the level separation of  $219\pm5$  keV indicates that the Doppler shifts of the two transitions are quite different. To investigate this point further, we measured the Doppler shift of the  $Be^{10}$  5.96  $\rightarrow 0$ transition at  $E_d = 3.1$  MeV and with the same target used in the present work. The measurement has been reported previously.<sup>27</sup> The result shows that the Doppler shift of the  $5.96 \rightarrow 0$  transition for the conditions of this experiment is  $31 \pm 1.2$  keV. Thus, the Doppler shift of the Be<sup>10</sup> 6.18-MeV level is  $(183\pm 6)$  $+(31\pm2)-(219\pm5)$  keV or  $-5\pm8$  keV.

From Eq. (19) the Doppler shift of the pair line, assuming the lifetime of the Be<sup>10</sup> 6.18-MeV level is short compared to the stopping time of the recoiling nuclei, is calculated to be 40 keV for an isotropic distribution of the recoiling Be<sup>10\*</sup> nuclei in the centerof-mass system and 24 keV if all the recoiling nuclei are emitted at 180° to the beam. Since the latter is the minimum possible shift the measured value of  $-5\pm8$  keV demands that the lifetime of the Be<sup>10</sup> 6.18-MeV level is not short compared to the stopping time of the recoiling nuclei and a lower limit for its lifetime can be obtained from the present results.

The stopping time ( $\alpha$ ) of the Be<sup>10</sup> nuclei in Be metal is estimated to be  $(5.4\pm0.5)\times10^{-13}$  sec.<sup>27</sup> To a good approximation, the observed Doppler shift is given<sup>27</sup> by  $F\Delta E_{\gamma}$  where  $\Delta E_{\gamma}$  is given by Eq. (19) and  $F = (\alpha/\tau)/2$  $(1+\alpha/\tau)$ , where  $\tau$  is the mean lifetime of the level. Combining the parameters given above, we find  $\tau < 5$  $\times 10^{-13}$  sec for the Be<sup>10</sup> 6.18-MeV level, where the limit is something like a 95% confidence limit. The Weisskopf estimates<sup>22</sup> of the mean lifetime of the Be<sup>10</sup>  $6.18 \rightarrow 0$  transition are  $0.9 \times 10^{-17}$  sec,  $1.3 \times 10^{-16}$  sec, and  $0.7 \times 10^{-13}$  sec for E1, M1, and E2 transitions, respectively (using a radius constant 1.2 F), while for the Be<sup>10</sup>  $6.18 \rightarrow 3.37$  transition they are  $0.98 \times 10^{-16}$ sec,  $1.4 \times 10^{-15}$  sec, and  $3.6 \times 10^{-13}$  sec, respectively. Thus, the lifetime of the Be<sup>10</sup> 6.18-MeV level is much longer than would be expected if it were J=1, 2 or 3, so that the information obtained on the lifetime and on the relative intensities of the  $6.18 \rightarrow 0$  and  $6.18 \rightarrow$ 3.37 transitions indicates that the Be<sup>10</sup> 6.18-MeV level is most probably  $J^{\pi}=0^+$  and the Be<sup>10</sup> 6.18  $\rightarrow 0$  transition an E0 transition. For a  $0^+$  assignment the Be<sup>10</sup> 6.18-MeV level can decay by an E1 transition to the 5.96-MeV level, and an E2 transition to the 3.37-MeV level. The most probable strengths of E1 and E2transitions in light nuclei, for the radius constant of 1.2 F, are  $\sim 0.055$  and  $\sim 5$  Weisskopf units, respectively. Using these values we estimate mean lifetimes for the E1 6.18  $\rightarrow$  5.96 and E2 6.18  $\rightarrow$  3.37 transitions of  $3.7 \times 10^{-12}$  sec and  $7 \times 10^{-13}$  sec, respectively. Without any other information we estimate that an E0 6.18  $\rightarrow$ 0 transition in Be<sup>10</sup> will most probably have a lifetime within a factor of 10 of the known C<sup>12</sup> and O<sup>16</sup> E0 lifetimes,<sup>11</sup> or about  $5 \times 10^{-11}$  sec. Thus, we predict that the order of magnitude of the 6.18  $\rightarrow$  0 transition branching ratio is 1%. This is in agreement with the limit obtained for the relative intensities of the 6.18  $\rightarrow$  0 and 6.18  $\rightarrow$  3.37 transitions, which becomes  $> 10^{-3}$  for an 0<sup>+</sup> assignment to the 6.18-MeV level.<sup>44</sup>

#### IV. COMPARISON WITH THE IPM

### A. The $B^{10}$ 3.58-MeV Level

This  $2^+$  state has been identified with an IPM state required at this excitation.<sup>28</sup> Our branching ratio of 4.2:1 for the transition to the first excited state of B<sup>10</sup> relative to that to the ground state may be compared with the IPM values shown in Table III, for the *M*1

TABLE III. IPM branching ratio R of the 3.58-MeV state of  $B^{10}$  to the first excited state relative to the ground state according to the IPM for M1 components alone and with inclusion of enhanced E2 components corresponding to an effective charge parameter x=0.5. (Experimentally: R=4.2.)

a/K R(M1) R(M1+E2)	0 0.00 0.17	$1 \\ 0.00 \\ 1.40$	$2 \\ 0.17 \\ 0.34$	$3 \\ 0.75 \\ 0.70$	$\begin{array}{c} 4 \\ 0.57 \\ 0.61 \end{array}$	5 0.44 0.49	$\begin{array}{c} 6 \\ 0.36 \\ 0.41 \end{array}$

components alone and also with the inclusion of enhanced E2 components corresponding to an effective charge parameter x=0.5 in the weak-coupling approximation (proton charge (1+x)e; neutron charge xe). Such an effective charge has been found necessary and adequate to give a phenomenological amount of the pure E2 transition to ground from the first excited states of Be<sup>10</sup> and B<sup>10</sup>.<sup>27</sup> (These and other computations of the predictions of the IPM given in this section have been made by J. M. Soper using the force mixture: W = 0.28, M = 0.45, B = 0.30, H = -0.03, and L/K = 6.We see that in the favored range 3-5 for the intermediate coupling parameter a/K the model fails by a factor of about 6-8 to give the right branching ratio. Only at  $a/K \approx 1$  is any sort of approach to the experimental ratio made. Such a low value of a/K is completely excluded by the level schemes of the nuclei A = 10 as has already been mentioned in this paper and elsewhere<sup>27</sup> and indeed the present demonstration that the 6.18-MeV level is  $0^+$  and not  $2^+$  seems to force a/K up towards 6 (see Sec. I).

A measurement of the absolute radiative width of this level would be of great interest.

<sup>&</sup>lt;sup>44</sup> The Be<sup>10</sup> 5.96- and 6.18-MeV levels have recently been definitely proven to be  $J^{\pi} = 1^{-}$  and 0<sup>+</sup>, respectively (Ref. 20).

### B. The $B^{10}$ 4.77-MeV Level

This 2<sup>+</sup> level (3<sup>+</sup> remains a less-likely possibility) is probably an interloper from the point of view of the IPM scheme.<sup>28</sup> We may pause to note that there is no temptation to identify it with the IPM level that we have just associated with the 3.58-MeV state since the experimental branching ratio of the 4.77-MeV state to the first excited state relative to the ground state is at least 10:1<sup>18,36</sup> (cf. Table III).

Our observation that  $\Gamma_{\alpha}/\Gamma_{\gamma} > 20$  enables us to give a value for  $\Gamma_{\gamma}$  from the earlier results on the  $(\alpha, \gamma)$ reaction.<sup>18,36</sup> If we accept  $J^{\pi} = 2^+$  we find  $\Gamma_{\gamma} \approx 0.03$  eV. The angular distributions indicate that the M1 and E2 components of the transition to the first excited state are of approximately equal strength, say 0.01–0.02 eV each. This corresponds to strengths in Weisskopf units of  $7-14 \times 10^{-3}$  for the M1 component and 9-17 for the E2 component (on a radius constant of 1.2 F). The weakness of the M1 transition is not surprising for an M1 transition between states of T=0 in a nucleus of  $T_z=0$  while the strong E2 transition mirrors the local systematics. We may note at this point that suggestions were made at the time of the "5.16-MeVstate puzzle," referred to above, that perhaps the 4.77-MeV state and not the 5.16-MeV state was the  $2^+$  T=1 analog of the first excited state of Be<sup>10</sup>. The suggestion was based on the fact that the first excited state transition of the 4.77-MeV state is the dominant one as also is that of the IPM  $2^+$  T=1 state (unlike the experimental situation for the 5.16-MeV state to which we refer shortly). This identification would have freed the 5.16-MeV state for use in solving the "puzzle." However, not only is the puzzle satisfactorily solved without this device but we now know (since  $\Gamma_{\alpha} \gg \Gamma_{\gamma}$ ) that the M1 absolute radiative width of the 4.77-MeV state is far too small for the identification with the  $2^+T = 1$  state (the IPM would then require  $\Gamma_{\gamma} \approx 1.1 \text{ eV}$ ). Yet another implicit demonstration that T=0 for the 4.77-MeV state is the strongly enhanced E2 width which we only expect between states of the same isotopic spin. In addition, of course, the 4.77-MeV state is excited in reactions that should lead to states of T=0 while the 5.16-MeV state is not. The groundstate transition from the 4.77-MeV level may be as weak as  $10^{-3}$  eV (if we take the lower limit on the branching ratio<sup>36</sup>). If  $J^{\pi} = 2^+$ , this represents an M1 strength of less than  $5 \times 10^{-4}$  Weisskopf units which is interestingly small. If we refer the limit to E2 radiation rather than M1, the corresponding strength is less than 0.4 Weisskopf units (or less than 0.3 units if  $J^{\pi} = 3^+$  is the correct choice). These values are interestingly low for E2 transitions without change of isotopic spin in the 1*p*-shell. This may reflect the non-appearance of the 4.77-MeV state in the IPM scheme and suggest some form of collective excitation based on the 0.72-MeV state. A good measurement of the ground-state transition would be valuable.

We may now examine the alpha-width of the 4.77-MeV level. Our limit on  $\Gamma_{\alpha}/\Gamma_{\gamma}$  implies  $\Gamma_{\alpha} > 0.6$  eV. The de-excitation to Li<sup>6</sup> is by *d*-wave alpha particles whether  $J^{\pi} = 2^+$  or  $3^+$ . If we use the standard radius  $R = 1.45(A_1^{2/3} + A_2^{2/3})$  F this corresponds to a reduced width  $\theta_{\alpha}^2 > 0.07$  (defined by  $\Gamma_{\alpha} = 2kP\theta_{\alpha}^2\hbar^2/MR$ ). This is a sufficiently large reduced width itself to exclude an assignment of T = 1 to this state, and to suggest that a measurement of  $\Gamma_{\alpha}$  may shed some light on the wave function of the B<sup>10</sup> 4.77-MeV state.

# C. The B<sup>10</sup> 5.16-MeV Level

This  $2^+$  T=1 state, the analog of the first excited state of Be<sup>10</sup>, has already received considerable discussion in this paper. We may first of all examine the gamma-ray branching ratios. We find relative transition probabilities to the ground, 0.72- and 2.15-MeV states of 0.055:0.295:0.65. These figures may be compared with the predictions of the IPM given in Table IV for the M1 components alone. For these  $\Delta T = 1$  transitions there is no enhancement of E2 transitions and the E2 widths are found to be negligible compared to the M1widths. We see that in the favored region  $a/K \approx 3-5$ there is no semblance of agreement with experiment. Only at  $a/K \approx 2$ , far too low a value from the point of view of the level schemes, is anything like agreement reached. As we have seen, it seems more likely than not, though it is not yet proved, that  $\Gamma_{\alpha}$  is somewhat larger than  $\Gamma_{\gamma}$ . If we make this assumption we can use the existing<sup>18</sup> measurements of gamma-ray yield in the  $(\alpha, \gamma)$  reaction to quote  $\Gamma_{\gamma 0} \approx 0.02 \text{ eV}$ ;  $\Gamma_{\gamma 0, 72} \approx 0.09$ eV;  $\Gamma_{\gamma^{2},15} \approx 0.19$  eV. Comparison with Table IV shows that in the region  $a/K \approx 3-5$  the IPM fails by more than an order of magnitude to account for the absolute radiative width, just as it failed badly on the branching ratios. In the region  $a/K \approx 2$  where some semblance of agreement on the branching ratios is found the predicted absolute width is still too high by a factor of several. Of course it may be that, despite the present indications from distorted-wave stripping theory,  $\Gamma_{\gamma}$  is in fact bigger than  $\Gamma_{\alpha}$  so the situation with respect to the absolute value of  $\Gamma_{\gamma}$  may change towards better agreement with the IPM. This makes rather interesting the determination of the particle cross-section ratios that will enable  $\Gamma_{\gamma}/\Gamma_{\alpha}$  to be derived from our present results or alternatively an approach to the  $\alpha$  versus  $\gamma$ branching ratio problem by another method.

For the purposes of discussing the alpha-particle width of the level assume  $\Gamma_{\alpha} \approx \Gamma_{\gamma}$ , i.e.,  $\Gamma_{\alpha} \approx 1.0$  eV. This corresponds to  $\theta_{\alpha}^2 \approx 8 \times 10^{-4}$ . This is a very reasonable figure for an alpha-particle transition inhibited by isotopic spin considerations in the present conditions of level spacing.<sup>45</sup>

<sup>&</sup>lt;sup>45</sup> D. H. Wilkinson, *Proceedings of the Rehovoth Conference on Nuclear Structure*, edited by H. J. Lipkin (North-Holland Publishing Company, Amsterdam, 1958), p. 175.

TABLE IV. IPM branching fractions F of the 5.16-MeV state of B<sup>10</sup> to the ground, 0.72- and 2.15-MeV states (normalized to the sum of these three transitions) for M1 components alone. (Experimentally:  $F_0=0.055$ ;  $F_{0.72}=0.295$ ;  $F_{2.15}=0.65$ .) IPM radiative widths  $\Gamma_{\gamma}$  in eV are shown in brackets.

a/K	0	1	2	3	4	5	6	
$F_0(M1) \\ F_{0,72}(M1) \\ F_{2.15}(M1)$	· · · · · · ·	$\begin{array}{c} 0.86(2.96) \\ 0.00(0.012) \\ 0.14(0.49) \end{array}$	$\begin{array}{c} 0.29(0.51) \\ 0.28(0.50) \\ 0.43(0.76) \end{array}$	$\begin{array}{c} 0.00(0.004) \\ 0.95(2.64) \\ 0.05(0.15) \end{array}$	$\begin{array}{c} 0.06(0.21) \\ 0.93(3.19) \\ 0.01(0.038) \end{array}$	$\begin{array}{c} 0.17(0.72) \\ 0.82(3.39) \\ 0.01(0.025) \end{array}$	$\begin{array}{c} 0.26(1.26) \\ 0.73(3.52) \\ 0.01(0.022) \end{array}$	

### D. The Be<sup>10</sup> 6.18-MeV Level

The work presented here makes it extremely likely that this level has  $J^{\pi}=0^+$  and this assignment has



FIG. 5. Résumé of the present results. Energies in MeV. The 0<sup>+</sup> and 1<sup>-</sup> assignment to the Be<sup>10</sup> 6.18- and 5.96-MeV levels are from Ref. 20, the other energy levels and spin-parity assignments are from previous work which is discussed in the text. Uncertain assignments are enclosed in parenthesis. The branching ratios (percents) are given under the assumption that for the levels in question there are no other decay modes than those which are shown. For the B<sup>10</sup> 3.58-MeV level decay the relative weights of the two higher energy transitions are given. In addition to these, the B<sup>10</sup> 3.58  $\rightarrow$  2.15 cascade, which was not looked for, is known to have a  $\sim 20\%$  branch (Ref. 11).

subsequently been made definite.<sup>20</sup> That this level may have these properties was suggested<sup>25</sup> on the basis of the configuration  $p^{4}2s^{2}$ . This assignment is also useful, as we have remarked in Sec. I, in showing that the second excited  $2^{+}$  state of Be<sup>10</sup> is at least as high in excitation as 7.54 MeV and so arguing for a high ( $\approx 6$ ) ratio for a/K rather than the very low values that seem to be asked for by the gamma transitions just discussed.

### **V. CONCLUSION**

The available information on the gamma decays of the bound Be<sup>10</sup> states and the B<sup>10</sup> 3.58- and 5.16-MeV levels is summarized in Fig. 5. The available information on the partial gamma widths of the three lowest  $\alpha$ -unstable states in B<sup>10</sup> is also shown.

We must conclude that the IPM gives a rather poor account of gamma-ray transitions in B<sup>10</sup>. We note that the theoretical results given here for the Soper force mixture are in quite good agreement with the results obtained by Kurath<sup>24</sup> with a rather different force mixture. Thus it does not appear likely that this poor performance of the IPM is due to the details of the calculation. Further study would be very useful, particularly if it could supply absolute widths and E2/M1 ratios for the low-lying states of B<sup>10</sup>.

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